FYJC - MATHEMATICS & STATISTICS

PAPER - I

CHAPTER 5 :

TRIGONOMETRIC FUNCTIONS

Compiled & Conducted @ JKSC

TRIGONOMETRIC FUNCTIONS

ELIMINATE ANGLE θ

01. $x = 2\cos\theta + 3\sin\theta$
$y = 2\sin\theta - 3\cos\theta$ **ans :** $x^2 + y^2 = 13$

02. $x\cos\theta + y\sin\theta = a$
$x\sin\theta - y\cos\theta = b$ **ans :** $x^2 + y^2 = a^2 + b^2$

03. $x = 3\sec\theta + 2\tan\theta$
$y = 2\sec\theta + 3\tan\theta$ **ans :** $x^2 - y^2 = 5$

04. $a\operatorname{cosec}\theta + b\cot\theta = p$
$a\cot\theta + b\operatorname{cosec}\theta = q$ **ans :** $a^2 - b^2 = p^2 - q^2$

05. $x = a\sec\theta + b\tan\theta$
$y = a\sec\theta - b\tan\theta$.
ans : $\left(\frac{x+y}{a}\right)^2 - \left(\frac{x-y}{b}\right)^2 = 4$

06. $p = a\operatorname{cosec}\theta + b\cot\theta$
$q = a\operatorname{cosec}\theta - b\cot\theta$.
ans : $\left(\frac{p+q}{a}\right)^2 - \left(\frac{p-q}{b}\right)^2 = 4$

07. $x = 2\sec\theta + 3\tan\theta$
$y = 3\sec\theta - 2\tan\theta$
ans : $(2x+3y)^2 - (3x-2y)^2 = 169$

08. $\tan\theta + \sin\theta = m$
$\tan\theta - \sin\theta = n$
Show that : $m^2 - n^2 = \pm\ 4\sqrt{mn}$

09. $\cot\theta + \cos\theta = p$
$\cot\theta - \cos\theta = q$
Show that : $p^2 - q^2 = \pm\ 4\sqrt{pq}$

10. $x = a\sin\theta + b\cos\theta$
$y = a\cos\theta - b\sin\theta$
Show that : $(ax - by)^2 + (bx + ay)^2 = (a^2 + b^2)^2$

11. $x = r\cos\theta.\cos\phi$, $y = r\cos\theta\sin\phi$, $z = r\sin\theta$
Show that : $x^2 + y^2 + z^2 = r^2$

01. $x = 2\cos\theta + 3\sin\theta$ $\quad x = 3\sin\theta + 2\cos\theta$

$y = 2\sin\theta - 3\cos\theta$ $\quad y = 2\sin\theta - 3\cos\theta$

Squaring

$$x^2 = 9\sin^2\theta + 12\sin\theta.\cos\theta + 4\cos^2\theta$$
$$y^2 = 4\sin^2\theta - 12\sin\theta.\cos\theta + 9\cos^2\theta$$

$$x^2 + y^2 = 13\sin^2\theta + 13\cos^2\theta$$
$$x^2 + y^2 = 13(\sin^2\theta + \cos^2\theta)$$
$$x^2 + y^2 = 13 \quad \ldots\ldots\ \theta \text{ eliminated}$$

02. $x\cos\theta + y\sin\theta = a$

$x\sin\theta - y\cos\theta = b$

Squaring

$$x^2\cos^2\theta + 2xy\cos\theta.\sin\theta + y^2\sin^2\theta = a^2$$
$$x^2\sin^2\theta - 2xy\cos\theta.\sin\theta + y^2\cos^2\theta = b^2$$

$$x^2(\sin^2\theta + \cos^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = a^2 + b^2$$
$$x^2 + y^2 = a^2 + b^2 \quad \ldots..\ \theta \text{ Eliminated}$$

03. $x = 3\sec\theta + 2\tan\theta$

$y = 2\sec\theta + 3\tan\theta$

Squaring

$$x^2 = 9\sec^2\theta + 12\sec\theta.\tan\theta + 4\tan^2\theta$$
$$\underset{-}{\ } y^2 = \underset{-}{\ }4\sec^2\theta \underset{-}{+} 12\sec\theta.\tan\theta \underset{-}{+} 9\tan^2\theta$$

$$x^2 - y^2 = 5\sec^2\theta - 5\tan^2\theta$$
$$x^2 - y^2 = 5(\sec^2\theta - \tan^2\theta)$$
$$x^2 + y^2 = 5 \quad \ldots\ldots\ \theta \text{ eliminated}$$

04. $a\operatorname{cosec}\theta + b\cot\theta = p$

$a\cot\theta + b\operatorname{cosec}\theta = q$

Squaring

$$a^2\operatorname{cosec}^2\theta + 2ab\operatorname{cosec}\theta.\cot\theta + b^2\cot^2\theta = p^2$$
$$\underset{-}{\ }a^2\cot^2\theta \underset{-}{+} 2ab\operatorname{cosec}\theta.\cot\theta \underset{-}{+} b^2\operatorname{cosec}^2\theta = \underset{-}{\ }q^2$$

$$a^2(\operatorname{cosec}^2\theta - \cot^2\theta) + b^2(\cot^2\theta - \operatorname{cosec}^2\theta) = p^2 - q^2$$
$$a^2(1) + b^2(-1) = p^2 - q^2$$
$$a^2 - b^2 = p^2 - q^2 \quad \ldots..\ \theta \text{ Eliminated}$$

05.

$$x = a\sec\theta + b\tan\theta$$
$$y = a\sec\theta - b\tan\theta$$
$$x + y = 2a\sec\theta$$
$$\sec\theta = \frac{x+y}{2a}$$

$$x = a\sec\theta + b\tan\theta$$
$$y = a\sec\theta - b\tan\theta \quad (-)$$
$$x - y = 2b\tan\theta$$
$$\tan\theta = \frac{x-y}{2b}$$

$$1 + \tan^2\theta = \sec^2\theta$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\left(\frac{x+y}{2a}\right)^2 - \left(\frac{x-y}{2b}\right)^2 = 1$$

.... θ eliminated

06.

$$p = a\,\mathrm{cosec}\,\theta + b\cot\theta$$
$$q = a\,\mathrm{cosec}\,\theta - b\cot\theta$$
$$p + q = 2a\,\mathrm{cosec}\,\theta$$
$$\mathrm{cosec}\,\theta = \frac{p+q}{2a}$$

$$p = a\,\mathrm{cosec}\,\theta + b\cot\theta$$
$$q = a\,\mathrm{cosec}\,\theta - b\cot\theta \quad (-)$$
$$p - q = 2b\cot\theta$$
$$\tan\theta = \frac{p-q}{2b}$$

$$1 + \cot^2\theta = \mathrm{cosec}^2\theta$$
$$\mathrm{cosec}^2\theta - \cot^2\theta = 1$$
$$\left(\frac{p+q}{2a}\right)^2 - \left(\frac{p-q}{2b}\right)^2 = 1$$

.... θ eliminated

07.

$$x = 2\sec\theta + 3\tan\theta \quad \times 2$$
$$y = 3\sec\theta - 2\tan\theta \quad \times 3$$
$$2x = 4\sec\theta + 6\tan\theta$$
$$3y = 9\sec\theta - 6\tan\theta$$
$$2x + 3y = 13\sec\theta$$
$$\sec\theta = \frac{2x+3y}{13}$$

$$x = 2\sec\theta + 3\tan\theta \quad \times 3$$
$$y = 3\sec\theta - 2\tan\theta \quad \times 2$$
$$3x = 6\sec\theta + 9\tan\theta$$
$$2y = 6\sec\theta - 4\tan\theta \quad (-)$$
$$3x - 2y = 13\tan\theta$$
$$\tan\theta = \frac{3x-2y}{13}$$

$$1 + \tan^2\theta = \sec^2\theta$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\left(\frac{2x+3y}{13}\right)^2 + \left(\frac{3x-2y}{13}\right)^2 = 1$$
$$(2x+3y)^2 + (3x-2y)^2 = 169$$

.... θ eliminated

08.

$$\tan\theta + \sin\theta = m$$
$$\tan\theta - \sin\theta = n$$

PROVE:

$$m^2 - n^2 = \pm 4\sqrt{mn}$$

$$\tan\theta + \sin\theta = m$$
$$\tan\theta - \sin\theta = n$$
$$2\tan\theta = m + n$$
$$\tan\theta = \frac{m+n}{2}$$
$$\cot\theta = \frac{2}{m+n}$$

$$\tan\theta + \sin\theta = m$$
$$\tan\theta - \sin\theta = n \quad (-)$$
$$2\sin\theta = m - n$$
$$\sin\theta = \frac{m-n}{2}$$
$$\mathrm{cosec}\,\theta = \frac{2}{m-n}$$

NOW

$$1 + \cot^2\theta = \mathrm{cosec}^2\theta$$
$$\mathrm{cosec}^2\theta - \cot^2\theta = 1$$
$$\frac{4}{(m-n)^2} - \frac{4}{(m+n)^2} = 1$$
$$4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$
$$4\left(\frac{(m+n)^2 - (m-n)^2}{(m-n)^2.(m+n)^2}\right) = 1$$
$$4\left[\frac{(m^2 + 2mn + n^2) - (m^2 - 2mn + n^2)}{[(m+n).(m-n)]^2}\right] = 1$$
$$4\left[\frac{m^2 + 2mn + n^2 - m^2 + 2mn - n^2}{(m^2 - n^2)^2}\right] = 1$$
$$4\,4mn = (m^2 - n^2)^2$$
$$16mn = (m^2 - n^2)^2$$
$$m^2 - n^2 = \pm 4\sqrt{mn} \quad \text{......... PROVED}$$

09. $\cot\theta + \cos\theta = p$

$\cot\theta - \cos\theta = q$

PROVE:

$p^2 - q^2 = \pm 4\sqrt{pq}$

$$\begin{aligned} \cot\theta + \cos\theta &= p \\ \cot\theta - \cos\theta &= q \\ \hline 2\cot\theta &= p + q \\ \cot\theta &= \frac{p+q}{2} \\ \tan\theta &= \frac{2}{p+q} \end{aligned}$$

$$\begin{aligned} \cot\theta + \cos\theta &= p \\ -\cot\theta \mp \cos\theta &= -q \\ \hline 2\cos\theta &= p - q \\ \cos\theta &= \frac{p-q}{2} \\ \sec\theta &= \frac{2}{p-q} \end{aligned}$$

NOW

$$1 + \tan^2\theta = \sec^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\frac{4}{(p-q)^2} - \frac{4}{(p+q)^2} = 1$$

$$4\ \frac{1}{(p-q)^2} - \frac{1}{(p+q)^2} = 1$$

$$4\left(\frac{(p+q)^2 - (p-q)^2}{(p-q)^2 . (p+q)^2}\right) = 1$$

$$4\ \frac{(p^2 + 2pq + q^2) - (p^2 - 2pq + q^2)}{[(p+q).(p-q)]^2} = 1$$

$$4\ \frac{p^2 + 2pq + q^2 - p^2 + 2pq - q^2}{(p^2 - q^2)^2} = 1$$

$$4\ 4pq = (p^2 - q^2)^2$$

$$16pq = (p^2 - q^2)^2$$

$p^2 - q^2 = \pm\ 4\ \sqrt{pq}$ ……….. PROVED

10. $x = a\sin\theta + b\cos\theta$

$y = a\cos\theta - b\sin\theta$ **Show that : $(ax - by)^2 + (bx + ay)^2 = (a^2 + b^2)^2$**

$x = b\cos\theta + a\sin\theta \quad \times a$

$y = a\cos\theta - b\sin\theta \quad \times b$

$$\begin{aligned} ax &= ab\cos\theta + a^2\sin\theta \\ -by &= -ab\cos\theta + b^2\sin\theta \\ \hline ax - by &= (a^2 + b^2)\sin\theta \end{aligned}$$

$x = b\cos\theta + a\sin\theta \quad \times b$

$y = a\cos\theta - b\sin\theta \quad \times a$

$$\begin{aligned} bx &= b^2\cos\theta + ab\sin\theta \\ ay &= a^2\cos\theta - ab\sin\theta \\ \hline bx + ay &= (a^2 + b^2)\cos\theta \end{aligned}$$

$$\sin\theta = \frac{ax - by}{a^2 + b^2} \qquad \cos\theta = \frac{bx + ay}{a^2 + b^2}$$

Now : $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{ax - by}{a^2 + b^2}\right)^2 + \left(\frac{bx + ay}{a^2 + b^2}\right)^2 = 1$$

$(ax - by)^2 + (bx + ay)^2 = (a^2 + b^2)^2$ ……… PROVED

11. $x = r\cos\theta.\cos\phi$, $y = r\cos\theta\sin\phi$, $z = r\sin\theta$

Show that : $x^2 + y^2 + z^2 = r^2$

LHS : $x^2 + y^2 + z^2$

$= r^2\cos^2\theta.\cos^2\phi + r^2\cos^2\theta\sin^2\phi + r^2\sin^2\theta$

$= r^2\cos^2\theta.(\cos^2\phi + \sin^2\phi) + r^2\sin^2\theta$

$= r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\cos^2\theta + \sin^2\theta)$

$= r^2$

= RHS

TRIGONOMETRIC FUNCTIONS

FIND ACUTE ANGLE θ & PERMISSIBLE

VALUES OF SIN X / COS X / TAN X

01. $2\cos A \tan B - 2\cos A - \tan B + 1 = 0$

ans : 60° ; 45°

02. $3\tan^2\theta - 4\sqrt{3}\tan\theta + 3 = 0$ **ans : 30° ;60°**

03. $3\cot^2\theta - 4\sqrt{3}\cot\theta + 3 = 0$ **ans : 30° ;60°**

04. $4\sin^2\theta - 2(\sqrt{3} + 1)\sin\theta + \sqrt{3} = 0$ **ans : 30° ;60°**

05. $4\cos^2\theta - 2(\sqrt{3} + 1)\cos\theta + \sqrt{3} = 0$ **ans : 30° ;60°**

06. $3(\operatorname{cosec}^2\theta + \cot^2\theta) = 5$ **ans : 60°**

07. $5\tan^2\theta + 3 = 9\sec\theta$ **ans : 60°**

08. $2\cos^2\theta + 3\cos\theta = 2$. Find $\cos\theta$ **ans : $1/2$**

09. $6\sin^2\theta - 11\sin\theta + 4 = 0$. Find $\sin\theta$ **ans : $1/2$**

10. $2\cos^2 x + 7\sin x = 5$. Find sin x **ans : $1/2$**

11. $\cot x + \operatorname{cosec} x = 5$; find cos x **ans : $12/13$**

12. $8\sin x - \cos x = 4$ **ans : $3/5$; $5/13$**

13. $\cos^2 x + 5\sin x.\cos x = 3$. Find tanx **ans :1 ; $2/3$**

01. $2\cos A \tan B - 2\cos A - \tan B + 1 = 0$

$2\cos A(\tan B - 1) - 1(\tan B - 1) = 0$

$(2\cos A - 1)(\tan B - 1) = 0$

$2\cos A - 1 = 0$ OR $\tan B - 1 = 0$

$\cos A = \frac{1}{2}$ OR $\tan B = 1$; $A = 60^\circ$ OR $B = 45^\circ$

02. $3\tan^2\theta - 4\sqrt{3}\tan\theta + 3 = 0$

$3\tan^2\theta - 3\sqrt{3}\tan\theta - 1\sqrt{3}\tan\theta + \sqrt{3}\sqrt{3} = 0$

$3\tan\theta(\tan\theta - \sqrt{3}) - \sqrt{3}(\tan\theta - \sqrt{3}) = 0$

$(3\tan\theta - \sqrt{3})(\tan\theta - \sqrt{3}) = 0$

$3\tan\theta - \sqrt{3} = 0$ **OR** $\tan\theta - \sqrt{3} = 0$

$\tan\theta = \frac{\sqrt{3}}{3}$; $\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

03. $3\cot^2\theta - 4\sqrt{3}\cot\theta + 3 = 0$

$3\cot^2\theta - 3\sqrt{3}\cot\theta - 1\sqrt{3}\cot\theta + \sqrt{3}\sqrt{3} = 0$

$3\cot\theta(\cot\theta - \sqrt{3}) - \sqrt{3}(\cot\theta - \sqrt{3}) = 0$

$(3\cot\theta - \sqrt{3})(\cot\theta - \sqrt{3}) = 0$

$3\cot\theta - \sqrt{3} = 0$ **OR** $\cot\theta - \sqrt{3} = 0$

$\cot\theta = \frac{\sqrt{3}}{3}$; $\cot\theta = \sqrt{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

04. $4\sin^2\theta - 2(\sqrt{3} + 1)\sin\theta + \sqrt{3} = 0$

$4\sin^2\theta - 2\sqrt{3}\sin\theta - 2\sin\theta + \sqrt{3} = 0$

$2\sin\theta(2\sin\theta - \sqrt{3}) - 1(2\sin\theta - \sqrt{3}) = 0$

$(2\sin\theta - 1)(2\sin\theta - \sqrt{3}) = 0$

$2\sin\theta - 1 = 0$ OR $2\sin\theta - \sqrt{3} = 0$

$\sin\theta = \frac{1}{2}$; $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$; $\theta = 60^\circ$

05. $4\cos^2\theta - 2(\sqrt{3} + 1)\cos\theta + \sqrt{3} = 0$

$4\cos^2\theta - 2\sqrt{3}\cos\theta - 2\cos\theta + \sqrt{3} = 0$

$2\cos(2\cos\theta - \sqrt{3}) - 1(2\cos\theta - \sqrt{3}) = 0$

$(2\cos\theta - 1)(2\cos\theta - \sqrt{3}) = 0$

$2\cos\theta - 1 = 0$ OR $2\cos\theta - \sqrt{3} = 0$

$\cos\theta = \frac{1}{2}$; $\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ$; $\theta = 30^\circ$

06. $3(\text{cosec}^2\theta + \cot^2\theta) = 5$

$3(1 + \cot^2\theta + \cot^2\theta) = 5$

$3(1 + 2\cot^2\theta) = 5$

$3 + 6\cot^2\theta = 5$

$6\cot^2\theta = 2$

$\cot^2\theta = \frac{2}{6}$

$\cot^2\theta = \frac{1}{3}$

$\cot\theta = +\frac{1}{\sqrt{3}}$ (θ is acute)

$\tan\theta = \sqrt{3}$, $\theta = 60^\circ$

07. $5\tan^2\theta + 3 = 9\sec\theta$

$5(\sec^2\theta - 1) + 3 = 9\sec\theta$

$5\sec^2\theta - 5 + 3 = 9\sec\theta$

$5\sec^2\theta - 2 = 9\sec\theta$

$5\sec^2\theta - 9\sec\theta - 2 = 0$

$5\sec^2\theta - 10\sec\theta + 1\sec\theta - 2 = 0$

$5\sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$(5\sec\theta + 1)(\sec\theta - 2) = 0$

$\sec\theta = \frac{-1}{5}$ OR $\sec\theta = 2$

$\cos\theta \neq -5$ $\quad$ $\cos\theta = \frac{1}{2}$

$(-1 \le \cos\theta \le 1)$ $\quad$ $\theta = 60^\circ$

08. $2\cos^2\theta + 3\cos\theta = 2$.

Find permissible values of $\cos\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

$2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$(2\cos\theta - 1)(\cos\theta + 2) = 0$

$2\cos\theta - 1 = 0$ OR $\cos\theta + 2 = 0$

$\cos\theta = \frac{1}{2}$ OR $\cos\theta \neq -2$

$(-1 \le \cos\theta \le 1)$

09. $6\sin^2\theta - 11\sin\theta + 4 = 0$.

Find $\sin\theta$

$6\sin^2\theta - 3\sin\theta - 8\sin\theta + 4 = 0$

$3\sin\theta(2\sin\theta - 1) - 4(2\sin\theta - 1) = 0$

$(3\sin\theta - 4)(2\sin\theta - 1) = 0$

$3\sin\theta - 4 = 0$ OR $2\sin\theta - 1 = 0$

$\sin\theta \neq \frac{4}{3}$ OR $\sin\theta = \frac{1}{2}$

$(-1 \le \sin\theta \le 1)$

10. $2\cos^2 x + 7\sin x = 5$. Find $\sin x$

$2(1 - \sin^2 x) + 7\sin x = 5$.

$2 - 2\sin^2 x + 7\sin x = 5$.

$2\sin^2 x - 7\sin x + 3 = 0$

$2\sin^2 x - 6\sin x - 1\sin x + 3 = 0$

$2\sin x(\sin x - 3) - 1(\sin x - 3) = 0$

$(2\sin x - 1)(\sin x - 3) = 0$

$\sin x = \frac{1}{2}$ OR $\sin x \neq 3$

$(-1 \le \sin x \le 1)$

11. $\cot x + \operatorname{cosec} x = 5$; find $\cos x$

$$\frac{\cos x}{\sin x} + \frac{1}{\sin x} = 5$$

$$\cos x + 1 = 5 \sin x$$

Squaring on both sides

$$(\cos x + 1)^2 = (5 \sin x)^2$$

$$\cos^2 x + 2\cos x + 1 = 25 \sin^2 x$$

$$\cos^2 x + 2\cos x + 1 = 25 (1 - \cos^2 x)$$

$$\cos^2 x + 2\cos x + 1 = 25 - 25\cos^2 x$$

$$26\cos^2 x + 2\cos x - 24 = 0$$

$$13\cos^2 x + \cos x - 12 = 0$$

$$13\cos^2 x + 13\cos x - 12\cos x - 12 = 0$$

$$13\cos x (\cos x + 1) - 12(\cos x + 1) = 0$$

$$(13\cos x - 12) (\cos x + 1) = 0$$

$$\cos x = \frac{12}{13} \quad \text{OR} \quad \cos x = -1$$

12. $8\sin x - \cos x = 4$

$$8\sin x - 4 = \cos x$$

$$(8\sin x - 4)^2 = \cos^2 x$$

$$64 \sin^2 x - 64 \sin x + 16 = 1 - \sin^2 x$$

$$65 \sin^2 x - 64 \sin x + 15 = 0$$

$$65 \sin^2 x - 39 \sin x - 25 \sin x + 15 = 0$$

$$13\sin x (5\sin x - 3) - 5(5\sin x - 3) = 0$$

$$(13\sin x - 5) (5\sin x - 3) = 0$$

$$\sin x = \frac{5}{13} \quad \text{OR} \quad \sin x = \frac{3}{5}$$

13. $\cos^2 x + 5\sin x.\cos x = 3$. Find $\tan x$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{5\sin x.\cos x}{\cos^2 x} = \frac{3}{\cos^2 x}$$

$$1 + 5 \frac{\sin x}{\cos x} = \frac{3}{\cos^2 x}$$

$$1 + 5 \tan x = 3\sec^2 x$$

$$1 + 5 \tan x = 3(1 + \tan^2 x)$$

$$1 + 5 \tan x = 3 + 3\tan^2 x$$

$$3\tan^2 x - 5 \tan x + 2 = 0$$

$$3\tan^2 x - 3\tan x - 2\tan x + 2 = 0$$

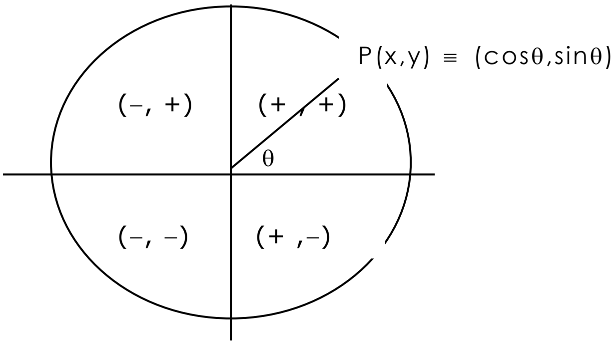
$$3\tan x (\tan x - 1) - 2(\tan x - 1) = 0$$

$$(3\tan x - 2)(\tan x - 1) = 0$$

$$\tan x = \frac{2}{3} \quad \text{OR} \quad \tan x = 1$$

TRIGONOMETRIC FUNCTIONS

ALL – SILVER – TEA – CUPS



I QUADRANT

cos θ & sin θ are positive

∴ tan θ is positive

Hence all ratios are positive

II QUADRANT

cos θ is –ve & sin θ is + ve

∴ tan θ is negative

Hence all ratios are negative except sin θ & cosec θ

III QUADRANT

cos θ & sin θ are negative

∴ tan θ is positive

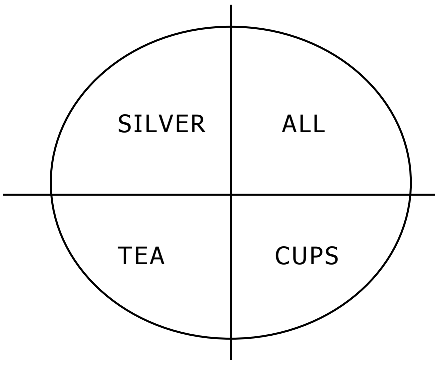
Hence all ratios are negative except tan θ & cot θ

IV QUADRANT

cos θ is +ve & sin θ is – ve

∴ tan θ is negative

Hence all ratios are negative except cos θ & sec θ



01. if $\cos\theta = 4/5$; $3\pi/2 < \theta < 2\pi$.

find all trignometric ratios

02. if $\cos\theta = 5/13$; $3\pi/2 < \theta < 2\pi$.

find all trignometric ratios

03. if $\tan\theta = 5/12$; $\pi < \theta < 3\pi/2$.

find all trignometric ratios

04. if $\tan\theta = -4/3$; $3\pi/2 < \theta < 2\pi$.

find $3\sec\theta + 5\tan\theta$

05. if $\cos\theta = -3/5$; $\pi < \theta < 3\pi/2$.

find $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$

06. $\dfrac{\sin A}{3} = \dfrac{\sin B}{4} = \dfrac{1}{5}$;

A and B are in II quadrants

find $4\cos A + 3\cos B$

07. if $\sec\theta = \sqrt{2}$; $3\pi/2 < \theta < 2\pi$.

find $\dfrac{1 + \tan\theta + \operatorname{cosec}\theta}{1 + \cot\theta - \operatorname{cosec}\theta}$

08. if $\cos A = \sin B = -1/3$;

where $\pi/2 < A < \pi$; $\pi < B < 3\pi/2$

find : $\dfrac{\tan A + \tan B}{\tan A - \tan B}$

09. $5\tan A = \sqrt{7}$; $\pi < A < 3\pi/2$.

$\sec B = \sqrt{11}$; $3\pi/2 < B < 2\pi$.

find $\operatorname{cosec}A - \tan B$

10. $2\sin x = 1$; $\pi/2 < x < \pi$;

$\sqrt{2}\cos y = 1$; $3\pi/2 < y < 2\pi$

find : $\tan x + \tan y$

$\overline{\cos x \ - \ \cos y}$

01. if $\cos\theta = 4/5$; $3\pi/2 < \theta < 2\pi$.

find all trignometric ratios

<u>SOLUTION</u>

θ lies in the IV Quadrant

$\therefore$ $\cos\theta$ & $\sec\theta$ are positive

$$\boxed{\cos\theta = \frac{4}{5} \ ; \quad \sec\theta = \frac{5}{4}}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \frac{16}{25} = 1$$

$$\sin^2\theta = 1 - \frac{16}{25}$$

$$\sin^2\theta = \frac{9}{25}$$

$$\boxed{\sin\theta = -\frac{3}{5} \ ; \quad \text{cosec} = -\frac{5}{3}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-3/5}{4/5}$$

$$\boxed{\tan\theta = -\frac{3}{4} \ ; \quad \cot\theta = -\frac{4}{3}}$$

02. if $\cos\theta = 5/13$; $3\pi/2 < \theta < 2\pi$.

find all trignometric ratios

<u>SOLUTION</u>

θ lies in the IV Quadrant

$\therefore$ $\cos\theta$ & $\sec\theta$ are positive

$$\boxed{\cos\theta = \frac{5}{13} \ ; \quad \sec\theta = \frac{13}{5}}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \ 25 \quad = 1$$

$\overline{169}$

$$\sin^2\theta = 1 - \frac{25}{169}$$

$$\sin^2\theta = \frac{144}{169}$$

$$\boxed{\sin\theta = -\frac{12}{13} \ ; \quad \text{cosec} = -\frac{13}{12}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-12/13}{5/13}$$

$$\boxed{\tan\theta = -\frac{12}{5} \ ; \quad \cot\theta = -\frac{5}{12}}$$

03. if $\tan\theta = 5/12$; $\pi < \theta < 3\pi/2$.

find all trignometric ratios

<u>SOLUTION</u>

θ lies in the III Quadrant

$\therefore$ $\tan\theta$ & $\cot\theta$ are positive

$$\boxed{\tan\theta = \frac{5}{12} \ ; \quad \cot\theta = \frac{12}{5}}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \frac{25}{144} = \sec^2\theta$$

$$\sec^2\theta = \frac{169}{144}$$

$$\boxed{\sec\theta = \frac{-13}{12} \ ; \quad \cos\theta = -\frac{12}{13}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{5}{12} = \frac{\sin\theta}{\frac{-12}{13}}$$

$$\sin\theta = \frac{5}{12} \times -\frac{12}{13} = -\frac{5}{12}$$

$$\boxed{\sin\theta = -5 \ ; \quad \text{cosec}\theta = -\underline{12}}$$

12   5

04. if $\tan\theta = -4/3$; $3\pi/2 < \theta < 2\pi$.

find $3\sec\theta + 5\tan\theta$

SOLUTION

θ lies in the IV Quadrant

$\therefore$ $\cos\theta$ & $\sec\theta$ are positive

$$\tan\theta = -\frac{4}{3} ; \quad \cot\theta = -\frac{3}{4}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \frac{16}{9} = \sec^2\theta$$

$$\sec^2\theta = \frac{25}{9}$$

$$\sec\theta = \frac{5}{3} ; \quad \cos\theta = \frac{3}{5}$$

NOW

$3\sec\theta + 5\tan\theta$

$$= 3 \cdot \frac{5}{3} + 5 \cdot \left(\frac{-4}{3}\right)$$

$$= 5 - \frac{20}{3}$$

$$= -\frac{5}{3}$$

05. if $\cos\theta = -3/5$; $\pi < \theta < 3\pi/2$.

find $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$

SOLUTION

θ lies in the III Quadrant

$\therefore$ $\tan\theta$ & $\cot\theta$ are positive

$$\cos\theta = -\frac{3}{5} ; \quad \sec\theta = -\frac{5}{3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \frac{9}{25} = 1$$

$$\sin^2\theta = 1 - \frac{9}{25}$$

$$\sin^2\theta = \frac{16}{25}$$

$$\sin\theta = -\frac{4}{5} ; \quad \operatorname{cosec}\theta = -\frac{5}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-4/5}{3/5}$$

$$\tan\theta = \frac{4}{3} ; \quad \cot\theta = \frac{3}{4}$$

NOW $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$

$$= \frac{-\frac{5}{4} + \frac{3}{4}}{-\frac{5}{3} - \frac{4}{3}}$$

$$= \frac{-\frac{2}{4}}{-\frac{9}{3}} = \frac{\frac{1}{2}}{3} = \frac{1}{6}$$

06. $\dfrac{\sin A}{3} = \dfrac{\sin B}{4} = \dfrac{1}{5}$;

A and B are in II quadrants

find $4\cos A + 3\cos B$

SOLUTION

A lies in the II Quadrant

$\therefore$ sin A & cosec A are positive

$$\sin A = \frac{3}{5}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\frac{9}{25} + \cos^2 A = 1$$

$$\cos^2 A = 1 - \frac{9}{25}$$

$$\cos^2 A = \frac{16}{25}$$

$$\cos A = -\frac{4}{5}$$

B lies in the II Quadrant

$\therefore$ sin B & cosec B are positive

$\sin B = \frac{4}{5}$

$\sin^2 B + \cos^2 B = 1$

$\frac{16}{25} + \cos^2 B = 1$

$\cos^2 B = 1 - \frac{16}{25}$

$\cos^2 B = \frac{9}{25}$

$\boxed{\cos B = -\frac{3}{5}}$

NOW : 4cosA + 3cosB

$= 4\left(\frac{-4}{5}\right) + 3\left(\frac{-3}{5}\right)$

$= -\frac{16}{5} - \frac{9}{5}$

$= -\frac{25}{5} \qquad = -5$

07. if $\sec\theta = \sqrt{2}$; $3\pi/2 < \theta < 2\pi$.

find $\frac{1 + \tan\theta + \operatorname{cosec}\theta}{1 + \cot\theta - \operatorname{cosec}\theta}$

SOLUTION

θ lies in the IV Quadrant

$\therefore$ $\cos\theta$ & $\sec\theta$ are positive

$\sec\theta = \sqrt{2}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \tan^2\theta = 2$

$\tan^2\theta = 1$

$\boxed{\tan\theta = -1 \ , \quad \cot\theta = -1}$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$1 + 1 = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta = 2$

$\boxed{\operatorname{cosec}\theta = -\sqrt{2}}$

Now : $\frac{1 + \tan\theta + \operatorname{cosec}\theta}{1 + \cot\theta - \operatorname{cosec}\theta}$

$= \frac{1 + (-1) + (-\sqrt{2})}{1 + (-1) - (-\sqrt{2})}$

$= \frac{1 - 1 - \sqrt{2}}{1 - 1 + \sqrt{2}}$

$= \frac{-\sqrt{2}}{+\sqrt{2}}$

$= -1$

08. if cos A = sin B = $-1/3$;

where $\pi/2 < A < \pi$; $\pi < B < 3\pi/2$

find : $\frac{\tan A + \tan B}{\tan A - \tan B}$

SOLUTION

A lies in the II Quadrant

$\therefore$ sin A & cosec A are positive

$\cos A = \frac{-1}{3}$

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A + \frac{1}{9} = 1$

$\sin^2 A = 1 - \frac{1}{9}$

$\sin^2 A = \frac{8}{9}$

$\boxed{\sin A = \frac{\sqrt{8}}{3}}$

$\tan A = \frac{\sin A}{\cos A}$

$= \sqrt{8}/3 \,/\, (-1/3)$

$\boxed{\tan A = -\sqrt{8}}$

B lies in the III Quadrant

$\therefore$ tan B & cot B are positive

$\sin B = -\frac{1}{3}$

$\sin^2 B + \cos^2 B = 1$

$\frac{1}{9} + \cos^2 B = 1$

$\cos^2 B = 1 - \frac{1}{9}$

$\cos^2 B = \frac{8}{9}$

$\boxed{\cos B = -\frac{\sqrt{8}}{3}}$

$\tan B = \frac{\sin B}{\cos B}$

$= -1/3 \,/\, (-\sqrt{8}/3)$

$\boxed{\tan B = \frac{1}{\sqrt{8}}}$

NOW : $\frac{\tan A + \tan B}{\tan A - \tan B}$

$= \frac{-\sqrt{8} + \frac{1}{\sqrt{8}}}{-\sqrt{8} - \frac{1}{\sqrt{8}}}$

$= \frac{-8+1}{-8-1}$

$= \frac{7}{9}$

09. $5\tan A = \sqrt{7}$; $\pi < A < 3\pi/2$.

$\sec B = \sqrt{11}$; $3\pi/2 < B < 2\pi$.

find $\operatorname{cosec} A - \tan B$

SOLUTION

A lies in the III Quadrant

$\therefore$ tan A & cot A are positive

$\tan A = \frac{\sqrt{7}}{5}$; $\cot A = \frac{5}{\sqrt{7}}$

$1 + \cot^2 A = \operatorname{cosec}^2 A$

$1 + \frac{25}{7} = \operatorname{cosec}^2 A$

$\operatorname{cosec}^2 A = \frac{32}{7}$

$\boxed{\operatorname{cosec} A = -\frac{4\sqrt{2}}{\sqrt{7}}}$

B lies in the IV Quadrant

$\therefore$ cos B & sec B are positive

$\sec B = \sqrt{11}$

$1 + \tan^2 B = \sec^2 B$

$1 + \tan^2 B = 11$

$\tan^2 B = 10$

$\boxed{\tan B = -\sqrt{10}}$

Now : $\operatorname{cosec} A - \tan B$

$= -\frac{4\sqrt{2}}{\sqrt{7}} - (-\sqrt{10})$

$= -\frac{4\sqrt{2}}{\sqrt{7}} + \sqrt{10}$

10. $2\sin x = 1$; $\pi/2 < x < \pi$;

$\sqrt{2}\cos y = 1$; $3\pi/2 < y < 2\pi$

find : $\dfrac{\tan x + \tan y}{\cos x - \cos y}$

SOLUTION

x lies in the II Quadrant

$\therefore$ sin x & cosec x are positive

$\sin x = \dfrac{1}{2}$

$\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{4} + \cos^2 x = 1$

$\cos^2 x = 1 - \dfrac{1}{4}$

$\cos^2 x = \dfrac{3}{4}$

$\boxed{\cos x = -\dfrac{\sqrt{3}}{2}}$

$\tan x = \dfrac{\sin x}{\cos x}$

$= \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$\boxed{\tan x = -\dfrac{1}{\sqrt{3}}}$

y lies in the IV Quadrant

$\therefore$ cosy & secy are positive

$\boxed{\cos y = \dfrac{1}{\sqrt{2}}}$

$\sin^2 y + \cos^2 y = 1$

$\sin^2 y + \dfrac{1}{2} = 1$

$\sin^2 y = 1 - \dfrac{1}{2}$

$\sin^2 y = \dfrac{1}{2}$

$\boxed{\sin y = -\dfrac{1}{\sqrt{2}}}$

$\tan y = \dfrac{\sin y}{\cos y}$

$= \dfrac{\frac{-1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$\boxed{\tan y = -1}$

Now

$\dfrac{\tan x + \tan y}{\cos x - \cos y}$

$= \dfrac{-\frac{1}{\sqrt{3}} + (-1)}{\frac{-\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}$

$= \dfrac{\frac{1}{\sqrt{3}} + 1}{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}$

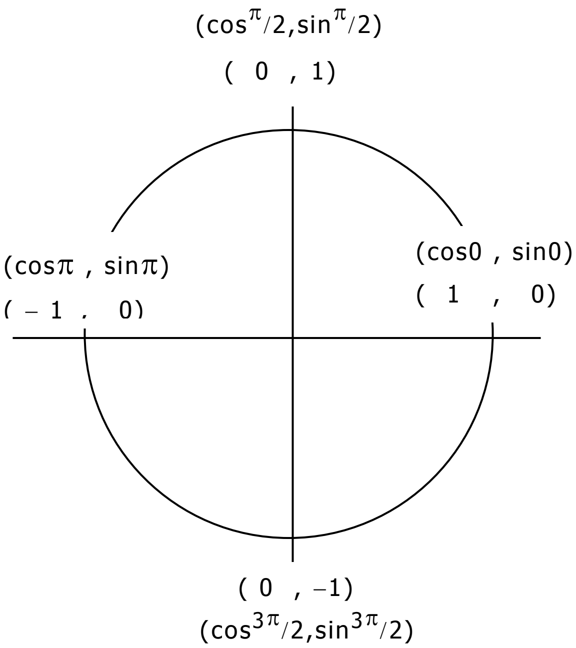
$= \dfrac{\frac{1 + \sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{6} + 2}{2\sqrt{2}}}$

$= \dfrac{1 + \sqrt{3}}{\sqrt{3}} \quad \dfrac{2\sqrt{2}}{\sqrt{6} + 2}$

$= \dfrac{1 + \sqrt{3}}{\sqrt{3}} \quad \dfrac{2\sqrt{2}}{\sqrt{2}(\sqrt{3} + \sqrt{2})}$

$= \dfrac{1 + \sqrt{3}}{\sqrt{3}} \quad \dfrac{2}{\sqrt{3} + \sqrt{2}}$

$= \dfrac{2(1 + \sqrt{3})}{3 + \sqrt{6}}$



30° – 45° – 60° – 90°

sin 30	=	cos 60	=	$^1/_2$
cos 30	=	sin 60	=	$^{\sqrt{3}}/_2$
sin 45	=	cos 45	=	$^1/_{\sqrt{2}}$
tan 45	=	cot 45	=	1
tan 60	=	cot 30	=	$\sqrt{3}$
tan 30	=	cot 60	=	$^1/_{\sqrt{3}}$

EVALUATE

01. $\sin^2 0 + \sin^2\left(\frac{\pi}{6}\right)^c + \sin^2\left(\frac{\pi}{3}\right)^c + \sin^2\left(\frac{\pi}{2}\right)^c$

02. $\cos^2 0 + \cos^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{2}\right)$

03. $\sin\pi + 2\cos\pi + 3\sin\left(\frac{3\pi}{2}\right) + 4\cos\left(\frac{3\pi}{2}\right)^c - 5\sec\pi - 6\,\text{cosec}\left(\frac{3\pi}{2}\right)^c$

04. $\sin 0 + 2\cos 0 + 3\sin\frac{\pi}{2} + 4\cos\frac{\pi}{2}^c + 5\sec 0 + 6\,\text{cosec}\,\frac{\pi}{2}^c$

05. $4\cot 45 - \sec^2 60 + \sin^2 30$

06. $\cot^2 60 + \sin^2 45 + \sin^2 30 + \cos^2 90$

07. $\sin^2 30 + \cos^2 60 + \tan^2 45 + \sec^2 60 - \text{cosec}^2 30$

08. $4\cot^2 30 + 9\sin^2 60 - 6\,\text{cosec}^2 60 - \frac{9}{4}\tan^2 60$

PROVE :

09 . $\dfrac{\sin^2\left(\frac{\pi}{3}\right) - \cos^2\left(\frac{\pi}{3}\right)}{\cos^2\left(\frac{\pi}{6}\right) \cdot \cos^2\left(\frac{\pi}{3}\right)} = \cot^2\frac{\pi}{6} - \tan^2\frac{\pi}{6}$

10. $\dfrac{\tan^2\left(\frac{\pi}{6}\right)^c + \sin^2\left(\frac{\pi}{6}\right)^c + \cos^2\left(\frac{\pi}{3}\right)^c}{\sec^2\left(\frac{\pi}{4}\right)^c - \cos^2\pi}$

$= \frac{1}{\sqrt{3}}\sec\left(\frac{\pi}{6}\right)^c + \frac{1}{3}\cos\left(\frac{\pi}{3}\right)^c$

01. $\sin^2 0 + \sin^2\left(\frac{\pi}{6}\right)^c + \sin^2\left(\frac{\pi}{3}\right)^c + \sin^2\left(\frac{\pi}{2}\right)^c$

$= \sin^2 0 + \sin^2 30 + \sin^2 60 + \sin^2 90$

$= 0 + \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= 0 + \frac{1}{4} + \frac{3}{4} + 1$

$= 2$

02. $\cos^2 0 + \cos^2\left(\frac{\pi}{6}\right)^c + \cos^2\left(\frac{\pi}{3}\right)^c + \cos^2\left(\frac{\pi}{2}\right)^c$

$= \cos^2 0 + \cos^2 30 + \cos^2 60 + \cos^2 90$

$= 1 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2$

$= 1 + \frac{3}{4} + \frac{1}{4} + 0$

$= 2$

03. $\sin \pi^c + 2\cos\pi^c + 3 \sin\left(\frac{3\pi}{2}\right)^c$
$+ 4 \cos\left(\frac{3\pi}{2}\right)^c - 5\sec \pi - 6 \text{ cosec}\left(\frac{3\pi}{2}\right)^c$

$= 0 + 2(-1) + 3(-1) + 4(0) - 5(-1) - 6(-1)$

$= 0 - 2 - 3 + 0 + 5 + 6$

$= 6$

04. $\sin 0 + 2 \cos 0 + 3 \sin \frac{\pi}{2}^c$
$+ 4 \cos \frac{\pi}{2}^c + 5 \sec 0 + 6 \text{ cosec } \frac{\pi}{2}^c$

$= 0 + 2(1) + 3(1) + 4(0) + 5(1) + 6(1)$

$= 0 + 2 + 3 + 0 + 5 + 6$

$= 16$

05. $4 \cot 45 - \sec^2 60 + \sin^2 30$

$= 4(1) - (2)^2 + \left(\frac{1}{2}\right)^2$

$= 4 - 4 + \frac{1}{4} \qquad = \frac{1}{4}$

06. $\cot^2 60 + \sin^2 45 + \sin^2 30 + \cos^2 90$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + 0$

$= \frac{1}{3} + \frac{1}{2} + \frac{1}{4}$

$= \frac{4 + 6 + 3}{12} \qquad = \frac{13}{12}$

07. $\sin^2 30 + \cos^2 60 + \tan^2 45 +$
$\sec^2 60 - \text{cosec}^2 30$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1 + (2)^2 - (2)^2$

$= \frac{1}{4} + \frac{1}{4} + 1 + 4 - 4$

$= \frac{1}{2} + 1 \qquad = \frac{3}{2}$

08. $4\cot^2 30 + 9\sin^2 60 - 6\text{cosec}^2 60$
$- \frac{9}{4} \tan^2 60$

$= 4(\sqrt{3})^2 + 9\left(\frac{\sqrt{3}}{2}\right)^2 - 6\left(\frac{2}{\sqrt{3}}\right)^2 - \frac{9}{4}(\sqrt{3})^2$

$= 12 + 9 \times \frac{3}{4} - 6 \times \frac{4}{3} - \frac{9}{4} \times 3$

$= 12 + \frac{27}{4} - 8 - \frac{27}{4}$

$= 4$

09 . $\dfrac{\sin^2\left(\frac{\pi}{3}\right) - \cos^2\left(\frac{\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3}\right) . \cos^2\left(\frac{\pi}{6}\right)} = \cot^2 \frac{\pi}{6} - \tan^2 \frac{\pi}{6}$

LHS $= \dfrac{\sin^2 60 - \cos^2 60}{\cos^2 60 . \cos^2 30}$

$= \dfrac{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2 . \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \dfrac{\frac{3}{4} - \frac{1}{4}}{\frac{1}{4} . \frac{3}{4}}$

$= \dfrac{\frac{1}{2}}{\frac{3}{16}}$

$= \dfrac{8}{3}$

RHS $= \cot^2 \frac{\pi}{6} - \tan^2 \frac{\pi}{6}$

$= \cot^2 30 - \tan^2 30$

$= (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 - \dfrac{1}{3}$

$= \dfrac{8}{3}$

LHS = RHS

10. $\dfrac{\tan^2\left(\frac{\pi}{6}\right)^c + \sin^2\left(\frac{\pi}{6}\right)^c + \cos^2\left(\frac{\pi}{3}\right)^c}{\sec^2\left(\frac{\pi}{4}\right)^c - \cos^2\pi}$

$= \dfrac{1}{\sqrt{3}} \sec\left(\frac{\pi}{6}\right)^c + \dfrac{1}{3} \cos\left(\frac{\pi}{3}\right)^c$

LHS

$= \dfrac{\tan^2 30 + \sin^2 30 + \cos^2 60}{\sec^2 45 - \cos^2 180}$

$= \dfrac{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}{(\sqrt{2})^2 - (-1)^2}$

$= \dfrac{\frac{1}{3} + \frac{1}{4} + \frac{1}{4}}{2 - 1}$

$= \dfrac{1}{3} + \dfrac{1}{2}$

$= \dfrac{2 + 3}{6}$

$= \dfrac{5}{6}$

RHS

$= \dfrac{1}{\sqrt{3}} \sec 30 + \dfrac{1}{3} \cos 60$

$= \dfrac{1}{\sqrt{3}} \dfrac{2}{\sqrt{3}} + \dfrac{1}{3} \dfrac{1}{2}$

$= \dfrac{2}{3} + \dfrac{1}{6}$

$= \dfrac{4 + 1}{6}$

$= \dfrac{5}{6}$

LHS = RHS

TRIGONOMETRIC FUNCTIONS

LHS = RHS

Q1.

01. $\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

02. $\sqrt{\dfrac{1 - \sin x}{1 + \sin x}} = \sec x - \tan x$

03. $\sqrt{\dfrac{\operatorname{cosec} x - 1}{\operatorname{cosec} x + 1}} = \dfrac{1}{\sec x + \tan x}$

04. $\sqrt{\dfrac{\sec x + 1}{\sec x - 1}} = \dfrac{1}{\operatorname{cosec} x - \cot x}$

Q2 .

01. $\dfrac{1}{1 - \sin \theta} + \dfrac{1}{1 + \sin \theta} = 2\sec^2\theta$

02. $\dfrac{\sin \theta}{1 + \cos \theta} + \dfrac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$

03. $\dfrac{\cos \theta}{1 - \sin \theta} + \dfrac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$

04. $\dfrac{\tan \theta}{\sec \theta + 1} + \dfrac{\sec\theta + 1}{\tan \theta} = 2 \operatorname{cosec} \theta$

05. $\dfrac{\cos \theta}{1 - \tan \theta} + \dfrac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$

06. $\dfrac{1}{\sec \theta - \tan \theta} - \dfrac{1}{\cos \theta}$

$= \dfrac{1}{\cos \theta} - \dfrac{1}{\sec \theta + \tan \theta}$

07. $\dfrac{1}{\operatorname{cosec} \theta - \cot \theta} - \dfrac{1}{\sin \theta}$

$= \dfrac{1}{\sin \theta} - \dfrac{1}{\operatorname{cosec} \theta + \cot \theta}$

08. $\dfrac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \dfrac{\sin \theta}{\cot\theta - \operatorname{cosec}\theta}$

Q3.

01. $\tan x + \cot x = \sec x \cdot \operatorname{cosec} x$

02. $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \cdot \operatorname{cosec}^2 x$

03. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

04. $\operatorname{cosec}^4 x - \operatorname{cosec}^2 x = \cot^4 x + \cot^2 x$

05. $\sin^4 x + \cos^4 x = 1 - 2\sin^2 x + 2\sin^4 x$

06. $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$

07. $\cos^6 A + \sin^6 A = 1 - 3\sin^2 A.\cos^2 A$

08. $\sec^6 x - \tan^6 x - 3\sec^2 x.\tan^2 x = 1$

09. $\operatorname{cosec}^6 x - \cot^6 x - 3\operatorname{cosec}^2 x.\cot^2 x = 1$

10. $\sin^3 x + \cos^3 x$
$= (\sin x + \cos x)(1 - \sin x \cos x)$

11. $\dfrac{\tan^3 x}{1 + \tan^2 x} + \dfrac{\cot^3 x}{1 + \cot^2 x}$
$= \sec x.\operatorname{cosec} x - 2\sin x, \cos x$

Q4 .

01. $\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \sin\theta - \cos\theta}{\sin\theta - 1 - \cos\theta}$

02. $\dfrac{\sin A}{1 - \cos A} = \dfrac{1 + \cos A - \sin A}{\sin A - 1 + \cos A}$

03. $\dfrac{1 + \sin A}{\cos A} = \dfrac{1 + \sin A + \cos A}{\cos A + 1 - \sin A}$

04. $\dfrac{\tan A}{\sec A - 1} = \dfrac{\tan A + \sec A + 1}{\sec A - 1 + \tan A}$

05. $\dfrac{1 + \operatorname{cosec} A + \cot A}{1 + \operatorname{cosec} A - \cot A}$
$= \dfrac{\operatorname{cosec} A + \cot A - 1}{-\operatorname{cosec} A + \cot A + 1}$

06. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

07. $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \dfrac{1 + \cos A}{\sin A}$

Q1.

01. $\sqrt{\dfrac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

LHS

$= \sqrt{\dfrac{1 - \cos A}{1 + \cos A} \; \dfrac{1 - \cos A}{1 - \cos A}}$

$= \sqrt{\dfrac{(1 - \cos A)^2}{1 - \cos^2 A}}$

$= \sqrt{\dfrac{(1 - \cos A)^2}{\sin^2 A}}$

$= \dfrac{1 - \cos A}{\sin A}$

$= \dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}$

$= \operatorname{cosec} A - \cot A = \text{RHS}$

02. $\sqrt{\dfrac{1 - \sin x}{1 + \sin x}} = \sec x - \tan x$

LHS

$= \sqrt{\dfrac{1 - \sin x}{1 + \sin x} \; \dfrac{1 - \sin x}{1 - \sin x}}$

$= \sqrt{\dfrac{(1 - \sin x)^2}{1 - \sin^2 A}}$

$= \sqrt{\dfrac{(1 - \sin x)^2}{\cos^2 x}}$

$= \dfrac{1 - \sin x}{\cos x}$

$= \dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}$

$= \sec x - \tan x = \text{RHS}$

03. $\sqrt{\dfrac{\operatorname{cosec} x - 1}{\operatorname{cosec} x + 1}} = \dfrac{1}{\sec x + \tan x}$

LHS

$= \sqrt{\dfrac{1 - \sin x}{1 + \sin x}}$

$= \sqrt{\dfrac{1 - \sin x}{1 + \sin x} \; \dfrac{1 - \sin x}{1 - \sin x}}$

$= \sqrt{\dfrac{(1 - \sin x)^2}{1 - \sin^2 x}}$

$= \sqrt{\dfrac{(1 - \sin x)^2}{\cos^2 x}}$

$= \dfrac{1 - \sin x}{\cos x}$

$= \dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}$

$= \sec x - \tan x \; \dfrac{\sec x + \tan x}{\sec x + \tan x}$

$= \dfrac{\sec^2 x - \tan^2 x}{\sec x + \tan x}$

$= \dfrac{1}{\sec x + \tan x}$

04. $\sqrt{\dfrac{\sec x + 1}{\sec x - 1}} = \dfrac{1}{\operatorname{cosec} x - \cot x}$

LHS

$= \sqrt{\dfrac{1 + \cos x}{1 - \cos x}}$

$= \sqrt{\dfrac{1 + \cos x}{1 - \cos x} \; \dfrac{1 + \cos x}{1 + \cos x}}$

$= \sqrt{\dfrac{(1 + \cos x)^2}{1 - \cos^2 x}}$

$= \sqrt{\dfrac{(1 + \cos x)^2}{\sin^2 x}}$

$$= \frac{1 + \cos x}{\sin x}$$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$= \operatorname{cosec} x + \cot x \ \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x}$$

$$= \frac{\operatorname{cosec}^2 x - \cot^2 x}{\operatorname{cosec} x - \cot x}$$

$$= \frac{1}{\operatorname{cosec} x - \cot x}$$

Q2 .

01. $\frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta} = 2\sec^2\theta$

$$= \frac{1 + \sin\theta + 1 - \sin\theta}{1 - \sin^2\theta}$$

$$= \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{RHS}$$

02. $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

$$= \frac{\sin^2\theta + (1 + \cos\theta)^2}{(1 + \cos\theta).\sin\theta}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1 + \cos\theta).\sin\theta}$$

$$= \frac{1 + 1 + 2\cos\theta}{(1 + \cos\theta).\sin\theta}$$

$$= \frac{2 + 2\cos\theta}{(1 + \cos\theta).\sin\theta}$$

$$= \frac{2(1 + \cos\theta)}{(1 + \cos\theta).\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta = \text{RHS}$$

03. $\frac{\cos\theta}{1 - \sin\theta} + \frac{1 - \sin\theta}{\cos\theta} = 2\sec\theta$

$$= \frac{\cos^2\theta + (1 - \sin\theta)^2}{(1 - \sin\theta).\cos\theta}$$

$$= \frac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{(1 - \sin\theta).\cos\theta}$$

$$= \frac{1 + 1 - 2\sin\theta}{(1 - \sin\theta).\cos\theta}$$

$$= \frac{2 - 2\sin\theta}{(1 - \sin\theta).\cos\theta}$$

$$= \frac{2(1 - \sin\theta)}{(1 - \sin\theta).\cos\theta}$$

$$= \frac{2}{\cos\theta}$$

$$= 2\sec\theta = \text{RHS}$$

04. $\frac{\tan\theta}{\sec\theta + 1} + \frac{\sec\theta + 1}{\tan\theta} = 2\operatorname{cosec}\theta$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + 1} + \frac{\frac{1}{\cos\theta} + 1}{\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta}$$ REFER 02

05. $\frac{\cos\theta}{1 - \tan\theta} + \frac{\sin\theta}{1 - \cot\theta} = \sin\theta + \cos\theta$

$$= \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}} + \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}}$$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta - \sin\theta}$$

$$= \cos\theta + \sin\theta$$

06. $\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\cos\theta}$

$$= \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}$$

WE PROVE

$$\frac{1}{\sec\theta - \tan\theta} + \frac{1}{\sec\theta + \tan\theta}$$

$$= \frac{2}{\cos\theta}$$

$$= \frac{\sec\theta + \tan\theta + \sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{2\sec\theta}{1}$$

$1 + \tan^2\theta = \sec^2\theta$

$$= \frac{2}{\cos\theta} = \text{RHS}$$

07. $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$

$$= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

WE PROVE

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= \frac{\operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta}$$

$$= 2\operatorname{cosec}\theta$$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$= \frac{2}{\sin\theta} = \text{RHS}$$

08. $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

WE PROVE

$$\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} - \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta} = 2$$

$$= \sin\theta\left(\frac{1}{\cot\theta+\operatorname{cosec}\theta} - \frac{1}{\cot\theta-\operatorname{cosec}\theta}\right)$$

$$= \sin\theta\left(\frac{\cot\theta - \operatorname{cosec}\theta - \cot\theta - \operatorname{cosec}\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}\right)$$

$$= \sin\theta\left(\frac{-2\operatorname{cosec}\theta}{-1}\right)$$

$$= \sin\theta \;\; 2.\frac{1}{\sin\theta}$$

$$= 2$$

Q3.

01. $\tan x + \cot x = \sec x \,.\, \operatorname{cosec} x$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \,.\, \cos x}$$

$$= \frac{1}{\sin x \,.\, \cos x}$$

$$= \sec x \,.\, \operatorname{cosec} x$$

02. $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \,.\, \operatorname{cosec}^2 x$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x, \sin^2 x}$$

$$= \frac{1}{\cos^2 x . \sin^2 x}$$

$$= \sec^2 x \,.\, \operatorname{cosec}^2 x$$

03. $\sec^4x - \sec^2x = \tan^4x + \tan^2x$

$= \sec^2x\,(\sec^2x - 1)$

$1 + \tan^2x = \sec^2x$

$= \sec^2x\,.\tan^2x$

$= (1 + \tan^2x)\,.\tan^2x$

$= \tan^2x + \tan^4x$

04. $\mathrm{cosec}^4x - \mathrm{cosec}^2x = \cot^4x + \cot^2x$

$= \mathrm{cosec}^2x\,(\mathrm{cosec}^2x - 1)$

$= \mathrm{cosec}^2x\,.\cot^2x$ ***$1 + \cot^2x = \mathrm{cosec}^2x$***

$= (1 + \cot^2x)\,.\cot^2x$

$= \cot^2x + \cot^4x$

05. $\sin^4x + \cos^4x = 1 - 2\sin^2x + 2\sin^4x$

$= \sin^4x + (\cos^2x)^2$

$= \sin^4x + (1 - \sin^2x)^2$

$= \sin^4x + 1 - 2\sin^2x + \sin^4x$

$= 1 - 2\sin^2x + 2\sin^4x$

06. $\sin^4x + \cos^4x = 1 - 2\cos^2x + 2\cos^4x$

$= (\sin^2x)^2 + \cos^4x$

$= (1 - \cos^2x)^2 + \cos^4x$

$= 1 - 2\cos^2x + \cos^4x + \cos^4x$

$= 1 - 2\cos^2x + 2\cos^4x$

07. $\cos^6A + \sin^6A = 1 - 3\sin^2A.\cos^2A$

$= (\cos^2A)^3 + (\sin^2A)^3$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (\cos^2A + \sin^2A)^3 - 3\cos^2A.\sin^2A(\cos^2A + \sin^2A)$

$= (1)^3 - 3\sin^2A.\cos^2A\,(1)$

$= 1 - 3\sin^2A.\cos^2A$

08. $\sec^6x - \tan^6x - 3\sec^2x.\tan^2x = 1$

WE PROVE

$\sec^6x - \tan^6x = 1 + 3\sec^2x.\tan^2x$

$= (\sec^2x)^3 - (\tan^2x)^3 - 3\sec^2x\tan^2x$

$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$= (\sec^2x - \tan^2x)^3 + 3\sec^2x\tan^2x(\sec^2x - \tan^2x)$

$= (1)^3 + 3\sec^2x\tan^2x(1)$

$= 1 + 3\sec^2x\tan^2x$

09. $\mathrm{cosec}^6x - \cot^6x - 3\mathrm{cosec}^2x.\cot^2x = 1$

WE PROVE

$\mathrm{cosec}^6x - \cot^6x = 1 + 3\mathrm{cosec}^2x.\cot^2x$

$= (\mathrm{cosec}^2x)^3 - (\cot^2x)^3 - 3\mathrm{cosec}^2x\cot^2x$

$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

$= (\mathrm{cosec}^2x - \cot^2x)^3 + 3\mathrm{cosec}^2x\cot^2x(\mathrm{cosec}^2x - \cot^2x)$

$= (1)^3 + 3\mathrm{cosec}^2x\cot^2x(1)$

$= 1 + 3\mathrm{cosec}^2x\cot^2x$

10. $\sin^3x + \cos^3x = (\sin x + \cos x)(1 - \sin x\cos x)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (\sin x + \cos x)\,(\sin^2x - \sin x.\cos x + \cos^2x)$

$= (\sin x + \cos x)\,(1 - \sin x.\cos x)$

11. $\dfrac{\tan^3 x}{1 + \tan^2 x} + \dfrac{\cot^3 x}{1 + \cot^2 x}$

$= \sec x.\operatorname{cosec} x - 2\sin x,\cos x$

$= \dfrac{\tan^3 x}{\sec^2 x} + \dfrac{\cot^3 x}{\operatorname{cosec}^2 x}$

$= \dfrac{\dfrac{\sin^3 x}{\cos^3 x}}{\dfrac{1}{\cos^2 x}} + \dfrac{\dfrac{\cos^3 x}{\sin^3 x}}{\dfrac{1}{\sin^2 x}}$

$= \dfrac{\sin^3 x}{\cos x} + \dfrac{\cos^3 x}{\sin x}$

$= \dfrac{\sin^4 x + \cos^4 x}{\cos x.\ \sin x}$

$= \dfrac{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x.\cos^2 x}{\cos x\ .\ \sin x}$

$= \dfrac{1 - 2\sin^2 x\ .\cos^2 x}{\cos x.\sin x}$

$= \dfrac{1}{\cos x.\sin x} - \dfrac{2\sin^2 x.\cos^2 x}{\cos x.\sin x}$

$= \sec x.\operatorname{cosec} x - 2\sin x.\cos x$

Q4.

01. $\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \sin\theta - \cos\theta}{\sin\theta - 1 - \cos\theta}$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = (1 - \cos\theta)(1 + \cos\theta)$

$\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$

By THEOREM OF EQUAL RATIOS

$\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \sin\theta - \cos\theta}{\sin\theta - 1 - \cos\theta}$

02. $\dfrac{\sin A}{1 - \cos A} = \dfrac{1 + \cos A - \sin A}{\sin A - 1 + \cos A}$

$\sin^2 A = 1 - \cos^2 A$

$\sin^2 A = (1 - \cos A)(1 + \cos A)$

$\dfrac{\sin A}{1 - \cos A} = \dfrac{1 + \cos A}{\sin A}$

By THEOREM OF EQUAL RATIOS

$\dfrac{\sin A}{1 - \cos A} = \dfrac{1 + \cos A - \sin A}{\sin A - 1 + \cos A}$

03. $\dfrac{1 + \sin A}{\cos A} = \dfrac{1 + \sin A + \cos A}{\cos A + 1 - \sin A}$

$\cos^2 A = 1 - \sin^2 A$

$\cos^2 A = (1 - \sin A)(1 + \sin A)$

$\dfrac{\cos A}{1 - \sin A} = \dfrac{1 + \sin A}{\cos A}$

By THEOREM OF EQUAL RATIOS

$\dfrac{1 + \sin A}{\cos A} = \dfrac{1 + \sin A + \cos A}{\cos A + 1 - \sin A}$

04. $\dfrac{\tan A}{\sec A - 1} = \dfrac{\tan A + \sec A + 1}{\sec A - 1 + \tan A}$

$1 + \tan^2 A = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$\tan^2 A = (\sec A - 1)(\sec A + 1)$

$\dfrac{\tan A}{\sec A - 1} = \dfrac{\sec A + 1}{\tan A}$

By THEOREM OF EQUAL RATIOS

$\dfrac{\tan A}{\sec A - 1} = \dfrac{\tan A + \sec A + 1}{\sec A - 1 + \tan A}$

05. $\dfrac{1 + \text{cosec } A + \cot A}{1 + \text{cosec } A - \cot A}$

$= \dfrac{\text{cosec } A + \cot A - 1}{-\text{cosec } A + \cot A + 1}$

$1 + \cot^2 A = \text{cosec}^2 A$

$\text{cosec}^2 A - \cot^2 A = 1$

$(\text{cosec}A - \cot A)(\text{cosec}A + \cot A) = 1$

$\dfrac{\text{cosec } A + \cot A}{1} = \dfrac{1}{\text{cosec } A - \cot A}$

By THEOREM OF EQUAL RATIOS

$\dfrac{\text{cosec}A + \cot A + 1}{1 + \text{cosec}A - \cot A} = \dfrac{\text{cosec}A + \cot A - 1}{1 - \text{cosec}A + \cot A}$

.... PROVED

06. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

RHS

$= \dfrac{1 + \sin\theta}{\cos\theta}$ .... (i)

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$ .... (ii)

$= \sec\theta + \tan\theta \times \dfrac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$

$= \dfrac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$

$= \dfrac{1}{\sec\theta - \tan\theta}$ .... (iii)

HENCE FROM (i) , (ii) & (iii)

$\dfrac{1 + \sin\theta}{\cos\theta} = \dfrac{\sec\theta + \tan\theta}{1} = \dfrac{1}{\sec\theta - \tan\theta}$

$= \dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

BY THEOREM OF EQUAL RATIOS

07. $\dfrac{\cot A + \text{cosec } A - 1}{\cot A - \text{cosec } A + 1} = \dfrac{1 + \cos A}{\sin A}$

$\dfrac{1 + \cos A}{\sin A}$ .... (i)

$= \dfrac{1}{\sin A} + \dfrac{\cos A}{\sin A}$

$= \text{cosec } A + \cot A$ ...... (ii)

$= \text{cosec } A + \cot A \times \dfrac{\text{cosec } A - \cot A}{\text{cosec } A - \cot A}$

$= \dfrac{\text{cosec}^2 A - \cot^2 A}{\text{cosec } A - \cot A}$

$= \dfrac{1}{\text{cosec } A - \cot A}$ .... (iii)

HENCE FROM (i) , (ii) & (iii)

$\dfrac{1+\cos A}{\sin A} = \dfrac{\text{cosec}A + \cot A}{1} = \dfrac{1}{\text{cosec}A - \cot A}$

$= \dfrac{\cot A + \text{cosec } A - 1}{\cot A - \text{cosec } A + 1}$

BY THEOREM OF EQUAL RATIOS

Q5.

01. if $\tan x + \cot x = 3$, then show that $\tan^4 x + \cot^4 x = 47$

$\tan x + \cot x = 3$

$(\tan x + \cot x)^2 = 9$

$\tan^2 x + 2\tan x . \cot x + \cot^2 x = 9$

$\tan^2 x + 2(1) + \cot^2 x = 9$

$\tan^2 x + \cot^2 x = 7$

Squaring

$(\tan^2 x + \cot^2 x)^2 = 49$

$\tan^4 x + 2\tan^2 x . \cot^2 x + \cot^4 x = 49$

$\tan^4 x + 2(1) + \cot^4 x = 49$

$\tan^4 x + \cot^4 x = 47$

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